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Estimation on Rational Speculative Bubbles in Stock Market by Using Generalised Johansen-Ledoit-Sornette Model

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Abstract

This paper discusses the generalized Johansen-Ledoit-Sornette (GJLS) model in determining and predicting the size of rational speculative bubbles in stock market. In this study, the GJLS model is used as an approach to detect the presence and prediction of rational speculative bubble sizes of the stock market. The stock markets chosen in this study are Hang Seng, Nikkei 225 and S&P 500. This is because China, Japan and United States are countries that pioneers the world economy. This paper presents step by step on finding the linear and non-linear parameters in the generalized Johansen-Ledoit-Sornette model using the Ordinary Least Squares method for the linear parameters and formulas for the non-linear parameters. There are 7 parameters that need to be identified which are A,B and C for linear parameter and ω, φ, β and t for the non-linear parameter. Then, the size of speculative rational bubbles is determined and predict using the GJLS model. The size of speculative rational bubbles for future cycle is forecasted. Then, the effect of the presence of rational speculative bubbles towards the economy is discussed.

Keywords: forecasting; generalized Johansen-Ledoit-Sornette; rational speculative bubbles; economic bubbles.

1 Introduction

Chaudhuri [2] stated that a stock is a form of securities that represents an ownership in a company. Stock trading allows a company to raise capital to pay off debt, launch new products and further expand on operations. Companies that are able to stand strong and have a strong influence can represent the stock market of a country. For investors, the stock offers the opportunity for investors to gain profit on the value of the stock and obtain dividend payments by the invested company. Stock prices affect investor and consumer confidence in the business, which in turn affects the overall economy. The relationship between the economy and the stock market is related. Both are capable of affecting the performance respectively. This shows that the stock market plays an important role in the economy of a country.

The presence of rational speculative bubbles is one factor that can lead to the collapse of a country's stock market and in turn, affect the economy of the country. According to [1], rational speculative bubbles are situations where the price of an asset exceeds its intrinsic value. The presence of rational speculative bubbles in economic and financial markets can cause economic downturn. Therefore, this paper is designed to look at the presence of rational speculative bubbles in the stock market and thus predict the future economic downturn.

This study was conducted to demonstrate the measures to identify linear and nonlinear parameters in the General Johansen-Ledoit-Sornette (GJLS) model and in turn shows the accuracy of rational speculative bubble size estimations according to the financial cycle that changes every 10 years [5]. The previous study conducted by [9] showed the methods that can be used in the derivation of the GJLS model. While the study from [4] shows the forecast for the fall years and the existence of financial crises as in 1999 and 2008.

Data for this study is taken from the Yahoo Finance website and are divided into five cycles. The data collected are stock market data from China, United States and Japan, namely Hang Seng, S&P 500 and Nikkei 225. The selection of these three stock markets is based on the influence of these countries on the global economic market [7]. The data were collected from 1965 to 2019. The data was then divided into five cycles, 1965 to 1975 for the first cycle, 1976 to 1985 for the second cycle, 1986 to 1997 for the third cycle, 1998 to 2008 for the fourth cycle and 2009 to 2018 for the fifth cycle. The data for 2019 are taken to identify intrinsic values for forecasting purposes.

The first cycle of the Hang Seng market has been taken since 1969. This is because China has been practicing open policy since 1969. The distribution of this cycle is based on the financial crisis and the period is based on a statement from [5], stating that the economic crisis occurs every 10 years. This research is the continuity of the previous study research paper which entitled Forecasting on the Collapse of Rational Speculative Bubble in Hang Seng, Nikkei 225 and S&P 500 in 2018 [3]. The previous study only used the second part of the GJLS model and in this study, the overall of the GJLS model is applied.

In this study, the size of the rational speculative bubble in the stock markets is identified for the second cycle to the fifth cycle. Then, the Mean Absolute Percentage Error (MAPE) of the result of the bubble size that have been identified and the actual size of rational speculative bubbles will be determined for each stock market. Lastly, the study forecasted the size of rational speculative bubbles for the future cycle which is the sixth cycle.

2 Generalised Johansen-Ledoit-Sornette Model (GJLS)

The GJLS model is used as an approach to detect the presence and prediction of rational speculative bubble sizes. According to [4], the GJLS model was built by Anders Johansen, Olivier Ledoit and Didier Sornette. The GJLS model is shown in the Equation (1):

$$p_t = p_1 + \exp A + B\left(t_c - t\right)^{\beta} 1 + C\cos\left(\omega \log\left(t_c - t\right) + \varphi\right). \tag{1}$$

In Equation (1), p_t is the stock price at time t, p_1 is the stock price at time $t = 1$ or also known as intrinsic value. A, B and C are linear parameters while ω, φ, β and t_c are non-linear parameters. In order to find the value of a linear parameter, the non-linear parameter must be identified first. The value of ω is determined by the three-peaked method described by [6]. This method of three consecutive peaks is performed by finding the fall peak on the stock price graph divided by the cycle as shown in Figure 1.

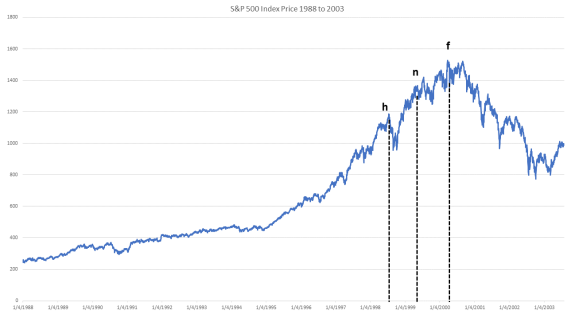

Figure 1: Finding the value of ω through three consecutive peaks, h, n and f.
Source : [6]

Figure 1 shows a graph of the index price for the S&P 500 market for the years 1988 to 2003. Figure 1 shows an illustration of finding three consecutive peaks denoted as h, n and f. h, n and f are the peak points before the sudden drop in price. In a cycle, there are more than one three consecutive peaks that exist. This means, there will be more than one value of ω for one cycle for each market. In order to determine the best value of ω, the size of the rational speculative bubble will be identified. The best value of ω is the value of ω that can provide the lowest MAPE value.

2.1 Linear & Non-linear Parameter

In Figure 1, the difference of three consecutive peaks is denoted as ρ. ρ is the parameter needed to find the value of ω. The value of ρ is identified by using Equation (2) and hence the value of ω can be searched through Equation (3).

$$\rho = (n - h) / (f - h), \tag{2}$$

$$\omega = 2\pi / \ln \rho. \tag{3}$$

If Figure 1 has five fall peaks. This means that there are three ω values to be gained. The more fall peaks are identified, the more ω values will be obtained. When ω values are identified, φ values are also obtained by Equation (4).

$$\phi = \phi - \omega \ln T - f; \text{ where } T = \frac{\rho f - j}{\rho - 1}. \tag{4}$$

2.2 Ordinary Least Square

The value of φ is based on the ω values obtained. The obtained ω and φ values were used to find the A, B and C values. Which means the values of parameters A, B and C obtained are based on ω and φ values. This means that the total value of A, B and C obtained is more than 1. According to the [8], the value of the linear parameter is obtained through Ordinary Least Square (OLS) Method. Equation (5) shows the model GJLS written in terms of f_t and g_t.

$$\ln p_t = A + Bf_t + Cg_t, \tag{5}$$

where $f_t = (t_c - t)^\beta$ and $g_t = (t_c - t)^\beta \cos(\omega \ln(t_c - t)\varphi)$. The estimation of the values of parameters A, B and C is solved using the OLS method as shown in Equation (6),

$$\sum_{t=t_1}^{t_n} \begin{pmatrix} \ln p_t \\ f_t \ln p_t \\ g_t \ln p_t \end{pmatrix} = \sum_{t=t_1}^{t_n} \begin{pmatrix} 1 & f_t & g_t \\ f_t & f_t^2 & f_t g_t \\ g_t & f_t g_t & g_t^2 \end{pmatrix} \begin{pmatrix} A \\ B \\ C \end{pmatrix}. \tag{6}$$

Equation (6) is rewritten in the form of matrix and is given as in Equation (7),

$$X^T y = (XTX)\,\beta. \tag{7}$$

The details of X, y and β values in Equation (7) are shown in Equation (8),

$$X = \begin{pmatrix} 1 & f_t 1 & g_t 1 \\ \vdots & \vdots & \vdots \\ 1 & f_t n & g_t n \end{pmatrix}, \; y = \begin{pmatrix} \ln p_t 1 \\ \vdots \\ \ln p_t n \end{pmatrix} \text{ and } \beta = \begin{pmatrix} A \\ B \\ C \end{pmatrix}. \tag{8}$$

Equation (8) can be solved using the Equation (9).

$$\hat{\beta} = (XTX)^{-1}XTy. \tag{9}$$

The values A, B and C are obtained using Equation (9). The obtained values will be more than 1. This is because, the values of ω and φ are more than 1. For example, through Figure 1, three values for each ω and φ are obtained, and this means three values of each parameter A, B and C.

2.3 Rational Speculative Bubbles

When the values of A, B and C are obtained, then the rational speculative bubble size can be identified through Equation (10), which is the second part of Equation (1). h_t is a notation of bubble size.

$$h_t = \exp A + B(t_c - t)^{\beta} \ \{1 + C\cos(\omega \log(t_c - t) + \varphi)\}. \tag{10}$$

The size of the bubbles obtained is based on the values of ω, φ, A, B and C. To determine the best value, MAPE testing is performed between the calculated bubble size and the actual bubble size. Actual bubble size values are obtained based on the differences between critical values, t_c and intrinsic values, a for each cycle in each stock market. The good parameter is the parameter that can determine the rational speculative bubble size with the lowest MAPE value.

Furthermore, the future forecasts of the rational speculative bubble size of each market are identified using the Equation (1). p_1 in Equation (1) is the intrinsic value, a, that is, the lowest peak value of a cycle. The results of this paper are discussed in the next section.

3 Result and Discussion

The empirical test results for the prediction of bubble sizes for second cycle through fifth cycle are shown in Table 1 through Table 3. The $h_n - 1$ notation used in the table shows the bubble size of the previous cycle that has been identified and the p_1 notation is the intrinsic value of current cycle.

Through Table 1 to Table 3, almost all MAPE value obtained is minimal. This shows the accuracy of the GJLS model in forecasting rational speculative bubble sizes. Therefore, the prediction of the rational speculative bubble size for the sixth cycle can be obtained by identifying the intrinsic value, p_1, for the sixth cycle.

Table 1: Forecasting Results on Rational Speculative Bubble Size for Hang Seng Market.

Hang Seng	h_{n-1}	p_1	Forecast	Actual	MAPE (%)
Second Cycle	1003.75	317.79	2007.5	2250.51	1.08
Third Cycle	1588.92	1894.9	3177.84	14778.4	7.85
Fourth Cycle	15223.8	6660.42	30447.6	24977.8	2.19
Fifth Cycle	17145.9	11344.6	34291.7	21809.5	5.72

Table 1 shows the forecasting results on rational speculative bubble size for Hang Seng market from the second cycle to the fifth cycle. From Table 1, the mean absolute percentage error is calculated between the forecast value and the actual value. The MAPE value for each cycle from Table 1 is less than 10 which indicate accurate forecasting results. So, from the MAPE value shows that the generalized Johansen-Ledoit-Sornette model can forecast the rational speculative bubbles for Hang Seng market.

Table 2: Forecasting Results on Rational Speculative Bubble Size for Nikkei 225 Market.

S&P 500	h_{n-1}	p_1	Forecast	Actual	MAPE (%)
Second Cycle	2967.68	4647.8	7615.48	14303.2	4.68
Third Cycle	11376	18544	29920	20372	4.69
Fourth Cycle	28310.6	7607.88	35918.5	10654.1	23.71
Fifth Cycle	16085.7	7054.98	23140.7	17069.2	3.56

Table 2 shows the forecasting results on rational speculative bubble size for Nikkei 225 market from the second cycle to the fifth cycle. From Table 2, the mean absolute percentage error is calculated between the forecast value and the actual value. The MAPE value for second, third and fifth cycles from Table 2 is less than 10 which indicate accurate forecasting results. While, the MAPE value for the fourth cycle is 23.71 which indicates a reasonable forecasting result. So, from the MAPE value shows that the generalized Johansen-Ledoit-Sornette model can forecast the rational speculative bubbles for Nikkei 225 market.

Table 3: Forecasting Results on Rational Speculative Bubble Size for S&P 500 Market.

S&P 500	h_{n-1}	p_1	Forecast	Actual	MAPE (%)
Second Cycle	113.91	86.9	200.81	167.1	2.02
Third Cycle	190.37	224.84	415.21	758.95	4.53
Fourth Cycle	871.27	776.76	1648.03	788.39	10.9
Fifth Cycle	1521.47	676.53	2198	2237.51	0.18

Table 3 shows the forecasting results on rational speculative bubble size for S&P 500 market from the second cycle to the fifth cycle. From Table 3, the mean absolute percentage error is calculated between the forecast value and the actual value. The MAPE value for second, third and fifth cycles from Table 3 is less than 10 which indicate accurate forecasting results. While, the MAPE value for the fourth cycle is 10.9 which indicates a good forecasting result. So, from the MAPE value shows that the generalized Johansen-Ledoit-Sornette model can forecast the rational speculative bubbles for S&P 500 market.

Results of the forecasting for rational speculative bubble sizes for the sixth cycle are shown in Table 4.

Table 4: Forecasting Results on Rational Speculative Bubble Size for Sixth Cycle.

Market	h_{n-1}	p_1	Forecast
Hang Seng	24358.8	25064.4	48717.5
Nikkei 225	19691.1	19562	39253.1
S&P 500	3000.96	2447.89	5448.85

Table 4 shows the forecasting results on rational speculative bubble size for the sixth cycle. The intrinsic value, p_1 for each market is identified based on the actual data for each stock market from January 2019 to April 2019. The h_{n-1} data is the size of rational speculative bubble that has been identified from the previous cycle (which is fifth cycle). The forecasted value from Table 4 indicated the size of rational speculative bubble for the sixth cycle (2019 to 2028) for each stock market. From the forecasted value, it is estimated that the rational speculative bubble will be burst when the size of bubbles approaching or exceeding the forecast value.

The empirical test results for this estimation indicate that the stock market is likely to fall when the bubble size value reaches or exceeds the forecast value obtained in each market as shown in Table 4. Compared to Table 1 to Table 3 and Table 4, the predicted bubble size for sixth cycle indicates that this bubble size is the highest compared to the bubble size of the other cycles for each market. If the economic downturn in 2008 was considered a severe fall, then the predicted fall for the sixth cycle would be worse than the fall in 2008 if viewed based on the predicted bubble

size.

4 Conclusion

The prediction accuracy of the rational speculative bubble size for the sixth cycle is based on the accuracy of the prediction results that have been carried out for first cycle through fifth cycle [3]. The analysis results for the prediction of rational speculative bubbles size, show that the results of the bubble size for the sixth cycle are in the range of accurate predictions to the reasonable predictions.

The results of forecasting the size of the bubble for the sixth cycle can be as a benchmark for investors and shareholders in investing activities. This is because, when the bubble size reaches the predicted size, the economic situation is likely to labile and the fall will probably be worse than the 2008 economic collapse.

These forecasts not only show that the GJLS model is capable on predicting the size of the rational speculative bubble but it also enables researchers to analyze the economic situation of the selected country in the presence of rational speculative bubbles.

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Conflicts of Interest The authors declare that no conflict of interests occurs.

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